On the Law of Rotation of the Sun. By H. F. Newall, F.R.S.

In the present note I desire to call attention to certain implications of the law of rotation, with which the Sun appears to try to conform, if we may judge the matter in the light of spectroscopic determinations.

When Carrington first formulated an empirical expression for the daily sidereal angular motion of sunspots in different latitudes about an axis of rotation and gave it the form

$$\xi = \mathrm{A} - \mathrm{B} \sin^{\frac{7}{4}} \phi,$$

Babinet pointed out (*Comptes Rendus,* **59**, 481) that for the sake of symmetry in the two hemispheres the power index of $\sin \phi$ should be an even integer; and Faye emphasised and developed this thesis.

Spectroscopists have sought in recent years to convince themselves that their observations conform satisfactorily with the allied relation

$$v + v_1 = (a - b \sin^2 \phi) \cos \phi \qquad . \quad . \quad . \quad (\mathrm{A})$$

where v is the velocity in the line of sight deduced from their actual observations; v_1 is a correction allowing for the orbital motion of the earth so as to convert synodic periods into sidereal; ϕ is the heliographic latitude; and a and b are adjustable velocities which have hitherto been regarded as constant from equator to pole. The question of the constancy of a and b *in time* throughout the eleven-year cycle has been freely discussed, but the evidence has hitherto been so conflicting, that it has not been possible to deduce any law of consistent variation in time.

The considerations which I now wish to set forth lead to a conclusion which I think deserves careful discussion. It is based on the idea that we have no reason to expect from a sun in tumultuous motion any great consistency in spectroscopic determinations. We should rather expect departures from a law of the kind implied in relation (A) than be surprised at them. If such departures had not been found, it would mean that the reversing layer must extend into regions either outwards, where turbulent motions cease and equilibrium is maintained by radiation and diffusion rather than by convective movements: or else inwards, where viscosity might be expected to preclude the possibility of violent motions.

I have been engaged for some years past in studying the outbursts and motions of sunspots as recorded in the Greenwich Photoheliographic Results, in the hope of being able to detect indications of local disturbances in what may be called the latitude-gradients of the daily sidereal motions of spots and spot-groups, $\dfrac{d\xi}{d\phi}$. It has been a troublesome research, but I hope to be able to present the very complex data in intelligible form before long; and this hope is strengthened by my having found helpful considerations suggested by a study of this relation (A)

$$v + v_1 = (a - b \sin^2 \phi) \cos \phi \, ;$$

and being desirous of setting forth these considerations prior to the meeting of the Astronomical Union in Rome in April next, I confine my attention in the present note to the spectroscopic problem.

The latitude-gradient of the velocity is

$$\frac{d(v+v_1)}{d\phi} = -\sin\phi(a+2b-3b\sin^2\phi).$$

It will be convenient to regard this as a positive retardation. If we inquire whether there can be a maximum in the value of the gradient in any latitude, we find

$$\frac{d^2(v+v_1)}{d\phi^2} = -\cos\phi(a+2b-9b\sin^2\phi);$$

and equating to zero, we get

$$\text{either } \phi = 90^\circ, \text{ or } \sin^2\phi = \frac{a+2b}{9b};$$

and inasmuch as the latter equation indicates the possibility of maxima depending on the relative values of a and b, we investigate it further, and find that it gives maxima in the sense that as we pass poleward the relative retardation of contiguous zones of latitude increases to a maximum at a latitude ϕ which depends largely on the value of b.

Now it appears from observation that a is fairly constant; and so we pay attention to b. We have for a maximum in the retardation

$$b = \frac{a}{9\sin^2\phi - 2}.$$

So long as we may regard a as having a value which varies, if at all, only within narrow limits, it is clear that we shall only get maxima in the values of $\frac{d(v+v_1)}{d\phi}$ if b has appropriate values. Table I. shows the values which b must have to give maxima in latitudes 90°, 80°, 70°, etc., consistent with equatorial velocities $a = 2{\cdot}10$ km./sec., $a = 1{\cdot}95$, and $a = 1{\cdot}80$.

TABLE I.

ϕ.	$b = \frac{a(=2{\cdot}10)}{9\sin^2\phi - 2}$.	$b = \frac{a(=1{\cdot}95)}{9\sin^2\phi - 2}$.	$b = \frac{a(=1{\cdot}80)}{9\sin^2\phi - a}$.
Pole 90°	·300 km./sec.	·278	·257
80	·312	·290	·267
70	·353	·328	·303
60	·440	·410	·379
50	·640	·594	·549
40	1·22	1·12	1·05
30	8·40	7·8	7·20
28·1	∞	∞	∞
20	neg.	neg.	neg.
10	,,	,,	,,
Equator 0	,,	,,	,,

We see, for instance, that if $b = \cdot 640$, there will be a maximum of retardation at $\phi = 50°$, if $a = 2\cdot 10$; or at lower values of ϕ if a is smaller than that value.

It is clear that if we release ourselves from the limitations involved in accepting only such numerical values of the constants a and b in the relation (A) as are required in the solar problem, then we should find that in order to get a maximum at latitudes lower than 40°, appropriate values of a and b can be assigned, including the passage of b through increasing finite values to infinity at $\phi = 28°\cdot 1$ ($\sin^2 \phi = \frac{2}{9}$). But these are of no interest in the solar problem.

We must remember that the relation (A) is an empirical expression accepted as expressing fairly closely the distribution of spectroscopic velocities in latitude; but it has not been based on any physical or hydrodynamical foundation, so far as I am aware. Babinet's even power of $\sin \phi$ is the only physical point put into it. Hence a careful scrutiny of the implications of the empirical law seemed desirable; and it has resulted in the finding of a clue which seems as if it may lead to a better understanding of some of the conditions underlying the circulation of the Sun's outer strata, and may serve to throw some light upon the relegation of sunspots to certain zones of latitude, and also possibly upon the latitude-movements of the spots and the prominences during the eleven-year cycle.

Though, for the solar problem, I have utilised the numerical mean values of a and b recently found by spectroscopists, in order to deduce the *position* of the maximum, and have in my lectures for some years past pointed out that these values place the maximum between latitudes 50° and 60°, I had not investigated the matter in detail.

But finding recently that Halm's observations in 1901 agreed well with Dunér's for that year in assigning the high values 0·70 and 0·79 to b, and further noting that the year 1901 saw the outburst of sunspots in high latitudes close upon 40° after the marked sunspot minimum, during which Dunér's observations had shown considerably smaller values of b, it occurred to me to revert to the matter; and Table I. is one of the first results. It seems to indicate that values of b higher than those commonly found by spectroscopists might possibly be forthcoming only at certain epochs in the sunspot cycle. And further examination of the observations supports the interesting conclusion that probably the values of both a and b are functions of the epoch in the sunspot cycle.

Spectroscopic observations of the solar rotation show that a has values generally well within the limits 2·15 to 1·85 km./sec., and that b has values ranging from about 0·300 to about 0·800 km./sec. and seldom approaches the value 1 km./sec. Thus we must infer from Table I. that the lowest latitude in which we can hope to see an organised maximum in the value of the gradient is about 40°, with a value b equal to about 0·6 or 0·8. We have to remember, however, that $d(v+v_1)/d\phi$ as here derived is the gradient of velocity for the gases on the spherical surface of the Sun, and as such it must be regarded as consisting of two parts, one arising from purely geometrical considerations due to the spherical shape, and the other arising from relative

motion of the gases on a sphere of reference. The choice of the proper equatorial velocity for a rotating *rigid* sphere of reference is dealt with in a later paragraph. It is enough here to state that maxima related to the maxima indicated in Table I. survive as maxima of retardation to be reckoned with. The fact that we do not usually find high values of b must be interpreted as indicating that there is something in the solar processes which forbids the corresponding distributions of velocities in contiguous zones of latitude between the equator and the latitude 40°.

We can, in fact, hardly resist the conclusion that with high values of b and of $-d(v+v_1)/d\phi$ the motion must change from stream-line motion to turbulent motion; and when the turbulence is organised (possibly by some form of precipitation or outburst), spots and vortices burst out near the latitude 40°.

When turbulence sets in, mixing of the contiguous zones at the latitude (say 40°) of the maximum gradient takes place with a reduction of the mean velocity. There follows a necessary increase of difference of velocity relative to that in the contiguous zone, say at latitude 39°, and a consequent increase of $d(v+v_1)/d\phi$ at 39°; and so the process of outburst of turbulence, accompanied here and there by spots and vortices, is gradually propagated towards the equator. On the other hand, in latitudes higher than 40° the relative motion of contiguous zones of latitude is diminished, and this tends to the persistence of stability polewards, in contradistinction to the propagation of instability equatorwards. Thus it would appear that there must be a gradual change (*in time*) of the gradient of velocity of the vapours, a change which is slowly propagated from the poles towards the equator, culminating after sunspot minimum in an outburst of sunspots about latitude 40°, with the subsequent convergence of the wide sunspot zones towards the equator in the eleven-year cycle. There is evidence that, when the spots have died out near the equator, the equatorial velocity increases; and this increase probably contributes to the width of the zone of instability between the equator and latitude 40°. Clearly a circulation between polar regions and equatorial regions involves movement of moment of momentum with very different effects, not only according to the direction in which it takes place, but also according to the level of the stratum in which the moment of momentum is carried.

This conception is, I take it, essentially that of Faye; but I am not aware that it has been elaborated in any detail in regard to the data now made available by the wealth of spectroscopic observations accumulated in the last twenty years. Before this was available, the problem of the solar rotation had been attacked by Harzer (*Ast. Nach.*, **127**, 17) and by Wilsing (*A.N.*, **127**, 333) and by Sampson (*Mem. R.A.S.*, **51**, 123) and by Wilczynsky (*Ap. J.*, **4**, 101; **7**, 124). Their work proceeds, however, chiefly in the study of the internal structure of the Sun; and it was impeded by uncertainty as to the gradients of density, pressure, temperature, and viscosity as we pass inwards into the body of the Sun. The effort of the present note is to extract the maximum amount of information from the mass of observational data relative to the actual motions of the vapours near the reversing layer I proceed first to give some tables, which serve to show schematic velocities and gradients.

Table II. exhibits the values of $v + v_1$ deduced for different values of b with an assumed value of a, viz. 2·10 km./sec.

Table II.

Velocities satisfying Relation (A), with a = 2·10 km./sec. and Various Values of b.

$b.$ / $\phi.$	0·00.	0·200.	0·250.	0·300.	0·350.	0·440.	0·640.	1·050.
0°	2·100	2·100	2·100	2·100	2·100	2·100	2·100	2·100
10	2·069	2·062	2·061	2·059	2·058	2·055	2·049	2·037
20	1·974	1·952	1·947	1·941	1·935	1·925	1·904	1·858
30	1·819	1·775	1·760	1·753	1·742	1·723	1·680	1·590
40	1·609	1·545	1·530	1·513	1·497	1·470	1·405	1·275 §
50	1·350	1·275	1·256	1·234	1·222	1·183	1·112 §	0·952
60	1·050	0·975	0·956	0·938	0·918	0·885 §	0·810	·656
70	0·718	·657	·643	·629	·612 §	·585	·523	·400
80	0·365	·332	·323	·315	·306	·291	·258	·180
90	0	0	0	0	0	0	0	0

§ Points of inflexion.

The column headed $b = 0·00$ gives the velocities for a solid or rigid sphere, which has an equatorial velocity of $a = 2·10$ km./sec. The tendency of these velocities with varying values of b is best seen in the graphs plotted in fig. 1. For low values of b, between 0·000 and 0·300, the latitude-velocity curves are concave towards the origin of co-ordinates. For higher values of b, points of inflexion appear in the curves. The positions of these points are indicated by § in Table II.

The inflected spindle or family of curves seen in fig. 1 shows the limits of the relation (A) for constant equatorial velocity of the vapours ($a = 2·10$) and various values of b. If both a and b vary at different epochs whilst they may be held to be constant from pole to equator at any one epoch, then the corresponding inflected spindles will move slightly on the diagram, with the polar end fixed at $\phi = 90°$, and the equatorial end moving through appropriate distances on the axis of velocities.

Now when we compare the spindles corresponding with $a = 2·10$ and $a = 1·90$, we see that between the latitudes 90° and 40° the curve ($a = 2·10$, $b = 1·00$) can scarcely be distinguished from the curve ($a = 1·90$, $b = 0·800$). There is no doubt about distinguishing between the equatorial ends of such curves.

We shall see in a later paragraph some justification for treating the observational data in at any rate two parts, one above latitude 40°, the other below that value. We shall see that the polar regions give velocities which are better fitted by the relation (A) with values of a and b different from those which best satisfy the equatorial regions

—a result which naturally follows from using four constants instead of two. [This procedure is an alternative to that sometimes adopted of inserting an additional (b, ϕ) term in the bracket of relation (A).] We need not lay undue stress on the extrapolated values of a derived from the data of the polar region.

The implication is that the circulation of the reversing vapours

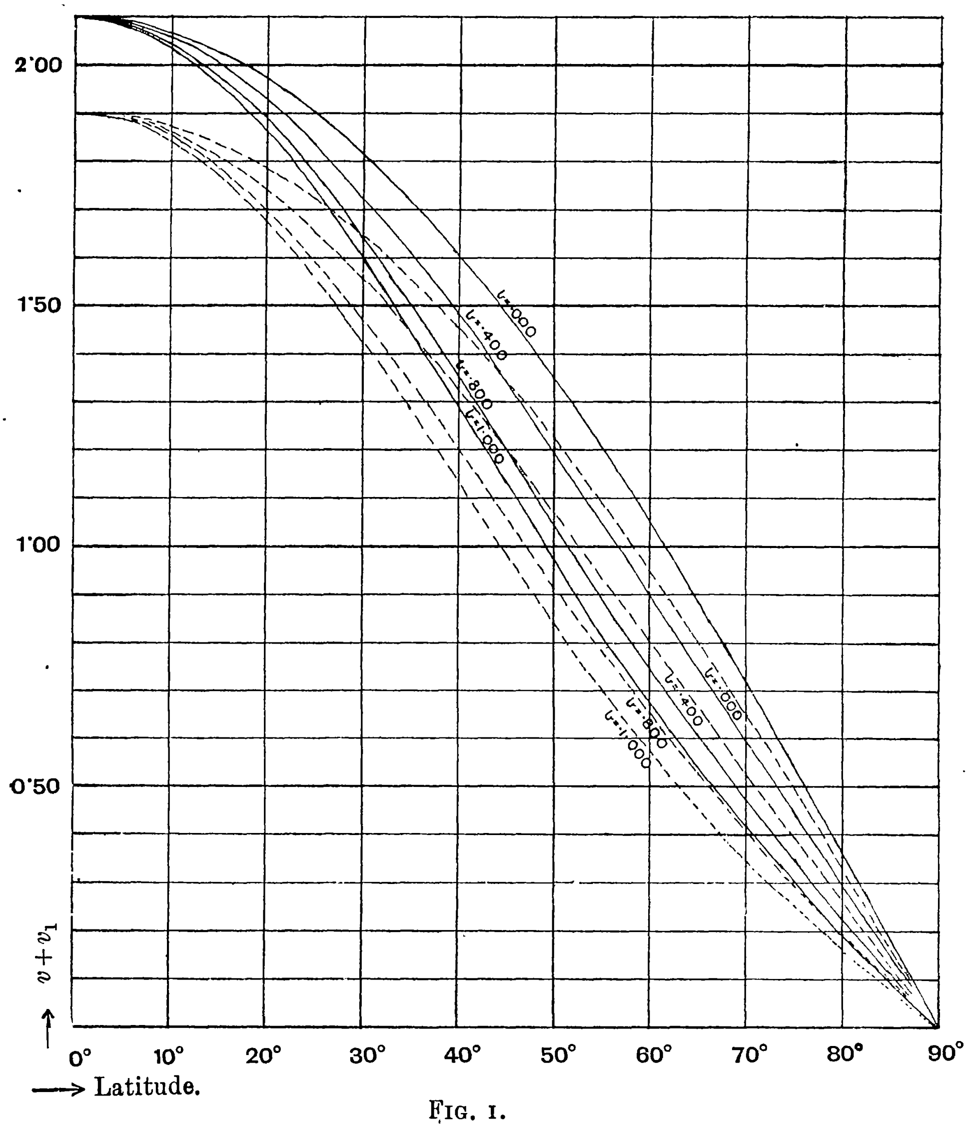

FIG. 1.

probably involves a movement of moment of momentum between the polar and equatorial regions: and evidence is adduced below to show that the velocities a and b are most probably functions of the phase of the eleven-year cycle of sunspot frequency.

Table III. exhibits the values of $-\dfrac{d(v+v_1)}{d\phi}$, the latitude-gradients of velocity in km./sec. per radian, for an assumed value of a, viz. 2·10 km./sec., and different values of b.

TABLE III.

Latitude-gradients of Velocity (Negative Sign omitted).

$b.$ / $\phi.$	0·00.	0·200.	0·250.	0·300.	0·350.	0·440.	0·640.	1·050.
0°	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
10	·365	·432	·448	·465	·482	·512	·578	·714
20	·718	·820	·859	·887	·916	·966	1·079	1·310
30	1·050	1·175	1·206	1·238	1·268	1·325	1·450	1·706
40	1·350	1·448	1·472	1·497	1·521	1·566	1·663	1·864*
50	1·609	1·645	1·655	1·664	1·673	1·689	1·726*	1·801
60	1·819	1·775	1·764	1·754	1·742	1·723*	1·680	1·591
70	1·974	1·852	1·821	1·790	1·760*	1·704	1·582	1·330
80	2·069	1·889	1·849	1·799	1·754	1·675	1·495	1·127
90	2·100	1·900	1·850	1·800	1·750	1·660	1·460	1·050

The asterisk * indicates maxima.
[For graphs of some of the figures in the table, see fig. 2 below.]

Table IV. exhibits the values of the gradients $-\dfrac{d(v+v_1)}{d\phi}$ expressed in metres per second per degree of latitude.

TABLE IV.

Latitude-gradients of Velocity (Metres per Second per Degree).

$b.$ / $\phi.$	0·00.	0·200.	0·250.	0·300.	0·350.	0·440.	0·640.	1·050.
0°	0	0	0	0	0	0	0	0
10	6·4	7·5	7·8	8·1	8·4	8·9	10·1	12·4
20	12·5	14·3	15·0	15·5	16·0	16·9	18·8	22·9
30	18·3	20·5	21·0	21·6	22·1	23·1	25·3	29·8
40	23·5	25·3	25·7	26·1	26·6	27·3	29·1	32·5*
50	28·1	28·7	28·9	29·1	29·2	29·5	30·1*	31·5
60	31·7	30·9	30·8	30·6	30·4	30·1*	29·3	27·8
70	34·5	32·3	31·8	31·2	30·7*	29·7	27·6	23·2
80	36·1	33·0	32·3	31·4	30 6	29·2	26·1	19·7
90	36·6	33·2	32·3	31·4	30·5	29·0	25·5	18·3

* Denotes a maximum.

The relation (A) gives the observed velocities of the vapours at different latitudes. It would be easy to compare these velocities with those on a rigid rotating sphere, if only we knew what angular velocity to give it. It seems natural to have recourse to some internal core with a large store of angular momentum as the obvious sphere of reference, from which we should deduce the latitude-changes in velocity of the vapours. The maintenance of the observed fairly constant velocity *a*

at the equator seems to demand some such store of angular momentum, unless we are to regard it as brought about by a struggle between an accelerator and a retarder, one inside, the other outside, the layers of vapours. In any case, it is clear that in dealing with the gradient $d(v+v_1)/d\phi$ we must be careful to discriminate between the geometrical part due to the spherical shape of the Sun and that part which constitutes the departure of the velocities of the vapours from the law of rotation of a rigid sphere.

If we take a rotating rigid nucleus with equatorial velocity A as the sphere of reference, the velocities due to the nucleus are given by $N = A \cos \phi$. The relative velocity of the vapours in latitude ϕ is

$$v + v_1 - N = (a - A - b \sin^2 \phi) \cos \phi.$$

This has zero values when $\phi = 90°$ and when

$$\sin^2 \phi = \frac{a - A}{b};$$

and if $a = A$, then $\phi = 0°$.

The gradient $\dfrac{d(v+v_1)}{d\phi} = \dfrac{dN}{d\phi}$, when $\sin^2 \phi = \dfrac{a - A + 2b}{3b}$;

and if $a = A$, $\sin^2 \phi = \frac{2}{3}$; consequently $\phi = 54°\cdot7$, and it is independent of the value of b.

This value of ϕ is of interest in connection with the distribution of prominences, and in particular with the phenomena of Deslandres' dark circumpolar filaments.

The value adopted for A is of special importance when we seek the proper interpretation of the maximum value of $d(v+v_1)/d\phi$, inasmuch as the part of the latter available for the explanation of the outburst of sunspots depends upon it. It seems probable that A must have a value which is equal to the highest observed value of a; and that smaller values of a may be produced by periodic wandering of vapours towards the equator from higher latitudes.

But it is clear that if we appeal to the maximum value of $\dfrac{d(v+v_1-N)}{d\phi}$ as the source of the production of instability in the relative motions of the gases, we must not overlook the shearing stresses between the vapours and the nucleus. An essential part of my present suggestion as to the origin of sunspots consists in taking account of the shearing stresses at the surface of the cone which has its apex at the Sun's centre and a semi-angle equal to the colatitude. These conical shears may lead to turbulent motion about axes always vertical to the Sun's surface, whatever latitude we may be dealing with.

On the other hand, the stresses introduced by the failure of the vapours to maintain the same velocity as the underlying nucleus lead to turbulence about axes far more nearly tangential to the surface of cylinders about the Sun's axis. These latter stresses are responsible for the vortices that have engaged the attention of Wilczynsky and Emden and others. It may be that the conical shears serve to turn the ends of Emden's vortices more nearly perpendicular to the surface of the Sun.

The values of Table III. are plotted in fig. 2 as a graph of gradients and latitudes. Here again, as in fig. 1, the curve for $a = 2 \cdot 10$ and $b = 0 \cdot 000$ is the graph for a rigid sphere rotating with equatorial velocity 2·10 km./sec. The difference between the ordinates for any given value of b and for $b = 0 \cdot 0$ gives the gradient in different latitudes of the vapours relative to the rigid sphere of reference ($a = 2 \cdot 10$), and we see

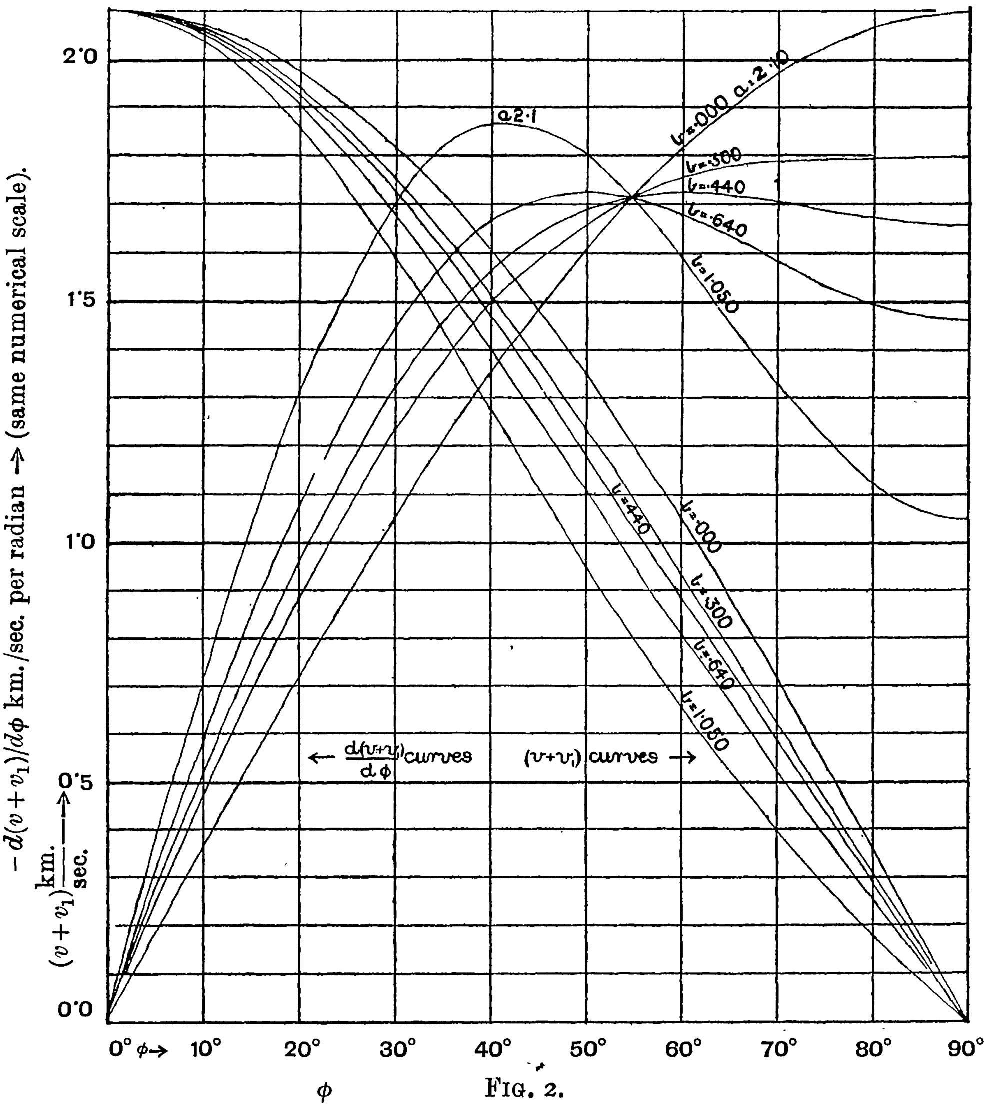

FIG. 2.

that the effective gradient of the velocity of the vapours is of the order of 10 metres per second per degree for the higher values of b.

Now the linear value of a degree on the Sun's apparent surface is about 12,000 km., and a wind of 100 km. per hour travels with a speed of 27·7 metres per second. Hence an observer stationed in latitude 30° on the sphere of reference ($a = 2 \cdot 10$) might find that, with $b = 0 \cdot 640$, the vapours 6000 km. to the south of him would be moving at a rate of about 25 km. per hour relative to the vapours 6000 km. to the north of him. The value of the kinematic viscosity (ν) which

would bring about instability would depend on the critical value of Vd/ν.

The viscosity with which we have here to deal is the kinematic viscosity $\nu = \eta/\rho$ of the vapours in the neighbourhood of the reversing layer, η being the ordinary (Poiseuille) coefficient of viscosity and ρ the density of the vapours. If we assume that the density in the reversing layer is of the order of $\frac{1}{10}$ of the normal density of atmospheric air, and further, that the ordinary viscosity η increases as the square root of the absolute temperature, then the kinematic viscosity in the reversing layer would be

$$\nu = \frac{10\eta_0}{\rho_0}\left(\frac{T}{273}\right)^{\frac{1}{2}},$$

when η_0 and ρ_0 are the values at 0° C. and 760 mm., and T is the absolute temperature of the reversing layer.

Now for hydrogen, $\eta_0 = 0{\cdot}84 \times 10^{-4}$, $\rho_0 = 0{\cdot}90 \times 10^{-4}$, so that $\frac{\eta_0}{\rho_0} = 0{\cdot}9 = \nu_0$ and $\nu_H = 10 \times 0{\cdot}9 \times \left(\frac{T}{273}\right)^{\frac{1}{2}}$.

(For mercury vapour, $\eta_0 = 1{\cdot}62 \times 10^{-4}$ at 0° C., if we accept Koch's value extrapolated from observations made at 300° and 380° C. This is apparently the only metallic vapour for which Poiseuille's coefficient has been determined.)

Again, $\left(\frac{T}{273}\right)^{\frac{1}{2}} = 5$ for T = 6825° Abs. C. : or 6 for T = 9820° Abs. C.

Hence for hydrogen in the reversing layer we have $\nu_H = 45$.

(For glycerine, $\eta_0 = 46{\cdot}0$, $\rho_0 = 1{\cdot}26$; $\nu = 36{\cdot}5$ at 0° C.)

These figures indicate a degree of viscidity which, while it argues well for effective rigidity in some layers close to the reversing layer, seems to preclude the possibility of the observed motions of the vapours outside it. The generally accepted view, that the pressure diminishes outwards as in a gravitational field (with $g = 27{,}000$ cm./sec.^{2}), involves a very rapid diminution of density in the vapours outside the reversing layer. Accordingly, the values of ν increase very rapidly from a prohibitive value in the reversing layer to still more prohibitive values outside it; and we are at a loss to explain the possibility of the observed signs of motion in the sunspot zones, the observed rapid motions of the prominences, and the clear indications of turbulent motions disclosed in the spectroheliograms of hydrogen. The observations forbid the rapid increase of ν. The only loophole of escape from the dilemma appears to lie in an admission that $d\rho/dr$ is much smaller than is usually allowed.

Discussion of Recent Spectroscopic Observations.

So far for *schematic* distributions of velocities and gradients. We pass now to a scrutiny of the actual results of spectroscopic observations. After the assiduous labours of Dunér, Halm, W. S. Adams, Plaskett, and many other observers, we have arrived at methods which give values of v of such precision that it is obvious that the observations deserve a confidence which is hardly yet admitted by the observers themselves.

The treatment of the observations usually employed is to accumulate observations of the velocity in the line of sight on many days at different solar latitudes; and, when a sufficient number has been secured, to treat them by the method of least squares for determining the most probable values of a and b in the adopted relation (A). The result is that we have found (unexpected?) variations in those values.

The considerations that I have now to urge tend to show that the secret lies in the fact that a least-square combination of observations at widely separated latitudes into *one* equation of the form of the relation

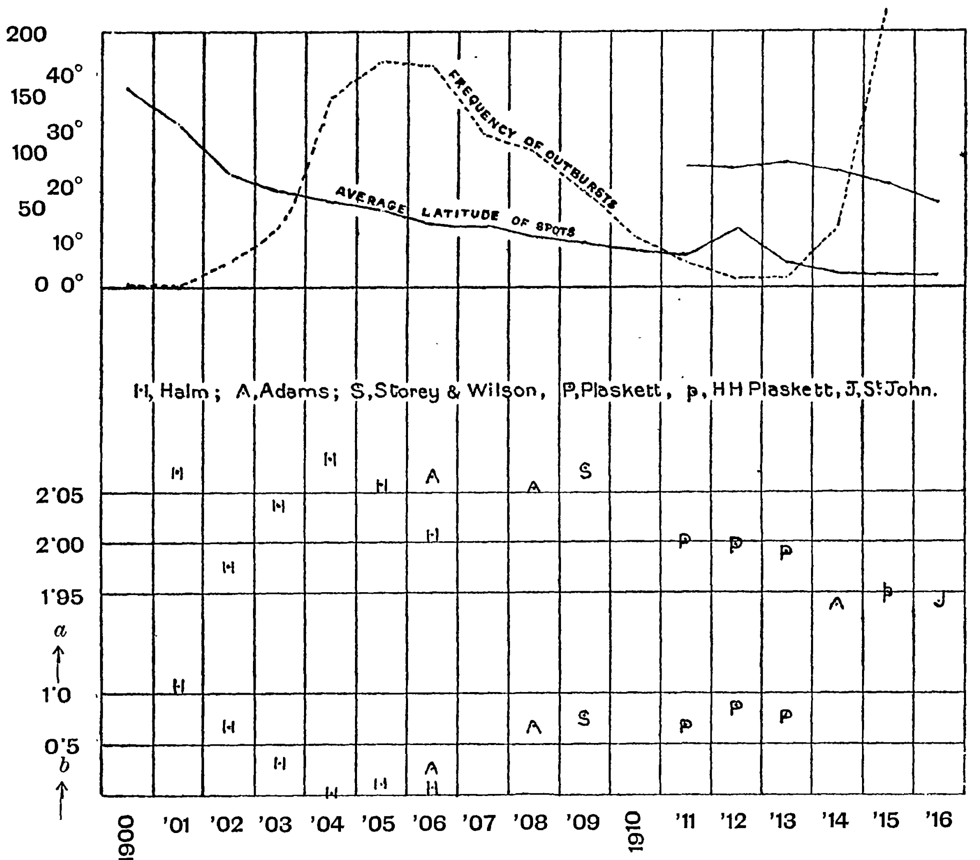

FIG. 3.

(A) is inadequate, by reason of real changes in the values of a and b at different latitudes—changes which are very probably a function of the phase of the eleven-year cycle of frequency of sunspots, or rather a function of the phase of an eleven-year cycle of changes in the whole mode of solar circulation, the sunspots being simply a symptom of this cycle of changes.

I deal with observations for every year in the sunspot cycle 1901–1913, except for the year 1910. Table V. exhibits the results of least-square solutions for each year. I have carried these out, firstly, with a combination of the spectroscopic velocities for all available latitudes into one equation of the form of relation (A); and the most probable values of the constants a and b are set forth in columns 3 and 4; these values are supposed to hold good from equator to pole. [It should be

stated that the observational data in most cases have been gathered by comparisons of the spectroscopic velocities at the limb of the Sun at points at the ends of solar diameters; thus they may include departures from regularity, which may in part be attributable to want of symmetry in the two hemispheres.]

The same observational data have next been divided into two parts, the one relative to latitudes 0° to 40° (equatorial half), the other relating to the latitudes 40° to 90° (polar half). Least-square solutions for a and b have then been carried out for these two parts independently, with the results shown in columns 5, 6, 7, and 8. The residuals (O – C) in nearly every year show (as is natural when four constants are utilised instead of only two) that the observations are better represented by values of a and b restricted to polar and to equatorial regions respectively.

It follows that the value of a deduced from the equatorial region approximates generally more closely to the observed equatorial velocity than the unrestricted value of a; and, on the other hand, the value of a deduced from the polar region is a somewhat wild extrapolation. This is indicated by enclosure in brackets in column 7.

The figures of Table V. are plotted in fig. 3 in graphs of a and time and b and time.

TABLE V.

Values of a and b derived by Weighted Least-square Solutions.

		All Latitudes.		Equatorial half, 0° to 40°.		Polar half, 40° to 90°.	
		a.	*b.*	*a.*	*b.*	*a.*	*b.*
1901	Halm	2·06	·70				
	N.	2·06	·74	2·07	·46	(2·40)	1·18
1902	Halm	1·973	·560				
	N.	1·975	·584	1·976	·518	(2·056)	·684
1903	Halm	2·036	·251				
	N.	2·041	·272	2·036	·264	(1·968)	·318
1904	Halm	2·075	·271				
	N.	2·054	·184	2·082	·337	(1·921)	·010
1905	Halm	2·039	·245				
	N.	2·027	·211	2·056	·392	(1·949)	·103
1906	Halm	2·010	·294				
	N.	2·006	·276	2·007	·199	(1·845)	·073
1907	Adams	2·055	·480				
	N.	...	...	2·065	·559	(1·919)	·262
1908	Adams	2·053	·546				
	N.	...	...	2·054	·592	(2·156)	·693
1909–1910	Storey & Wilson	...	...				
	N.	2·088	·496	2·074	·433	(2·237)	·792
1911	Plaskett	2·012	·500				
	N	...	...	2·003	·415	(2·156)	·673
1912	Plaskett	2·002	·558				
	N.	...	...	1·999	·529	(2·210)	·870
1913	Plaskett	1·993	·518				
	N.	...	...	1·991	·479	(2·161)	·788

I have hesitated to draw a mean curve through the values of a and of b, for I have had some doubt as to whether the observations of Halm which deal with the Dunér lines at 6302 are quite comparable with the observations of Adams and Plaskett which relate to many more lines in more extended regions in the solar spectrum. I regret that I have not been able to find published accounts of detailed observations for the war-years 1914–1919; it is a pity that these are lacking as yet, but when they are available it will be interesting to see whether the indications of the graphs of fig. 3 are confirmed by observations which cover the whole of an eleven-year cycle. It may be that we have then to exclude the earlier values till 1906, and to lay greater stress on the more modern observations, which constitute more homogeneous material.

I must allow that the arguments adduced above partake of the nature of groping for light in a somewhat disappointingly obscure subject. If, however, we are to lay any weight on the considerations which I set forth, it would appear that the relation (A) should be replaced by an expression of the form

$$v + v_1 = \left[a_0 \left\{ 1 + \frac{c}{a_0} \cos\left(\frac{2\pi(t - t_0)}{11 \cdot 13} + \epsilon_1 \right) \right\} - b_0 \left\{ 1 + \frac{d}{b_0} \cos\left(\frac{2\pi(t - t_0)}{11 \cdot 13} + \epsilon_2 \right) \right\} \sin^2 \phi \right] \cos \phi,$$

where a_0 and b_0 are constants, mean values of the variable a and b of relation (A) throughout the eleven-year cycle; c and d are constants, semi-amplitudes of the total observed variation of a and b; t the time (in years) from some fixed epoch t_0 in the 11·13-year cycle of sunspots; and ϵ_1 and ϵ_2 constants to adjust the phases of the variation of a and b to the chosen epoch. In the treatment of the observations it would seem to be desirable to deduce the values of a and b for the equatorial and polar regions by separate least-square solutions. The values roughly indicated are $a_0 = 2{\cdot}00$, $b_0 = 0{\cdot}400$, $c = 0{\cdot}6$, $d = 0{\cdot}4$. It would be rash to assign the relative values of ϵ_1 and ϵ_2 or to choose t_0 from data now available. If we try to form an idea of the cycle of changes in the solar circulation that appear to be indicated by the observational data of rotational velocity, the following may serve as a first attempt. In it I enter on the cycle at the epoch of minimum frequency of sunspots.

At sunspot minimum the vapours on the Sun at about latitude 50° lag behind those in contiguous zones of latitude with maximum retardation ($b = 0{\cdot}7$ or $0{\cdot}8$). The equatorial vapours are then moving with their lowest speed ($a = 1{\cdot}94$ or $1{\cdot}95$), but they are then beginning (with the disappearance of spots at the equator) to regain speed (possibly by reason of the tangential stresses between them and the effectively solid nucleus below them). The critical relative velocity between contiguous zones of latitude is reached somewhere below latitude 40° with outburst of spots in those latitudes (*e.g.* in 1901). The changes that follow are most tersely summarised in tabular form.

	Spots.	Velocities of Equatorial Region.	Velocities in Polar Regions 40° to 90°.	Prominences.
1900	At minimum			
1901	Burst out at Lat. 40°	$a'=1{\cdot}95$ Low velocity	$b'=0{\cdot}8$ Low velocities	Max. at 50° between 47° and 62°
1902	Lats. 27° to 18°	Increasing	$b'=0{\cdot}7$ Increasing	,, 52 ,, 52 ,, 70
1903	Lats. 28° to 12°	Increasing	$b'=0{\cdot}3$ Increasing	,, 52 ,, 52 ,, 70
1904	Lats. 30° to 5°	$a'=2{\cdot}00$ Increasing	$b'=0{\cdot}0$ Steady	,, 60 ,, 60 ,, 70
1905	Lats. 35° to 4° At maximum	$a'=2{\cdot}05$ Increasing	$b'=0{\cdot}1$ Falling	,, 68 ,, 68 ,, 80
1906	Lats. 27° to 3°	$a'=2{\cdot}07$ Steady. High	$b=0{\cdot}3$ Falling	,, 72 ,, 72 ,, 90
1907	Lats. 26° to 2°	..	..	Polar prominences die out at poles
1908	Lats. 23° to 2°	$a'=2{\cdot}07$ Steady. High	$b'=0{\cdot}7$ Falling	
1909	Lats. 17° to 2°	$a'=2{\cdot}07$(?) Falling	$b'=0{\cdot}8$ Steady. Low	
1910	Lats. 16° to 2°	..		
1911	Lats. 11° to 1°	$a'=2{\cdot}00$ Falling	$b=0{\cdot}8$ Steady. Low	
1912	Lats. 21° to 9°	..	$b=0{\cdot}8$ Steady. Low	
1913	At minimum, none near 0°	$a'=1{\cdot}99$ Falling	$b'=0{\cdot}8$ Steady. Low	
1914	Burst out at Lats. 32° to 19°, none at 0°	$a'=1{\cdot}95$ Steady. Low	? Increasing	

The dark filamentous flocculi recorded on so many recent spectroheliograms have special interest in several ways. (*a*) They are regarded as prominences seen by absorption as dark regions on the brighter background of the disc of the Sun. If their pressure is low and their absorptive power is consequently only large for wave-lengths in the immediate vicinity of their monochromatic emission, their apparent ability to cut out the light from the background must be due to some property other than absorptive power. Can it be that they appear dark by reason of their diverting the light of the background by refracting it in various directions? (*b*) Their enormous length compared with their width is a striking feature, not exactly what one would expect from eruptive matter spreading in higher regions of the Sun's envelope. (*c*) Their capricious movements sometimes towards sunspots, sometimes in apparent disregard of sunspots or even in repulsion from spots, suggests that they must consist sometimes of negatively electrified ions, sometimes of positively electrified ions; and if such ions are organised in huge vortices, then their capricious movements would receive explanation partly as a result of a conflict between the actions of the general magnetic field of the Sun and of the powerful local fields discovered by Hale in sunspots. It is not easy to explain their long-continued persistence without the disruption that is to be expected in vortices of ions of the same sign of electrification. It would be interesting to know whether a spot is from the moment of its outburst always the seat of a magnetic field or only when one of these dark filaments has been drawn into it. If such a filament were really a vortex of definite electrical sign and in definite mechanical rotation, it would go a long way to remove one of the difficulties connected with the explanation of the origin of magnetic fields in the sunspots, viz. the separation of the ions of opposite signs in a purely mechanical vortex. The valuable studies of Deslandres, Hale, Evershed and Royds have shown us that these dark filaments may serve to indicate the directions of a circulation in the higher regions of the Sun's envelopes.

On a Proposal of a New Method of securing Spectroscopic Observations of the Rotation of the Sun. By H. F. Newall, F.R.S.

A sphere viewed from a distance looks like a circular disc. If the sphere is rotating about an axis, the position of the projected axis may be represented by a straight line drawn across the apparent disc in the appropriate position angle. A chord drawn on the disc parallel to the projected axis may be regarded as representing the edge-on-view of a small circle AC on the sphere cut by a plane whose central normal is perpendicular to the axis and also to the direction of view. It is easy to show that if the sphere were rotating like a rigid body about the actual axis and were viewed from a point so distant that the *variation* in the inclination of the line of sight to the axis may be neglected, then the motion in the line of sight of every point in the area of the small circle AC is the same, for any given inclination of the axis to the line of sight; and clearly the motion in the line of sight of every point in the area of a second small circle BD similarly situated on the other side of the axis of rotation has the same numerical value though of opposite sign.

It has occurred to me that it may be well to utilise a corollary of this property of a rotating rigid sphere in determining by spectroscopic observations the law of rotation of the Sun, which is known to be rotating according to a complex law different (at any rate on what we call the surface of the Sun) from the simple law of a rotating rigid sphere. We should then in a sense reduce the problem to a determination of the law of departure of solar rotation from rigid rotation.

Utilising the terms equator, latitude, longitude, and central meridian in their usual signification, we see that if two chords AC, BD are taken parallel to the rotation axis NS and equidistant from it on opposite sides, we may measure with the spectroscope the velocity-shifts for pairs of points P and Q equidistant from the equatorial diameter EW. Deducing (for the conditions of solar presentation obtaining at the time of observation) the latitude of the points P and Q and the longitude measured from the central meridian, we can obtain the solution of our problem in a way which allows us to study the law of rotation in each hemisphere of the Sun separately. For purposes of convenient description we may call these equal parallel chords (AC, BD) "the velocity chords."

According to the particular object of our research, we must choose the velocity chords suitably; *e.g.* for the purpose of search for change in rotational velocity in the spot-bearing zones between the equator and the latitudes $\pm 40°$, we may take as velocity chords the chords passing from North to South through points on the limb midway between the equator and the axial points on the limb; or again, if we wish specially to study the rotational velocities in high latitudes, we must take the velocity chords much closer together.

For the sake of keeping the number of observations within reasonable limits, it is well to calculate beforehand the linear distances of P and Q from the equatorial diameter EW, for definite latitudes such as 0°, 10°, 20°, 30°, 40°, or 60°, 70°, 80°, 85°. In this way we shall accumulate